COMPARATIVE ANALYSIS OF NUMERICAL METHODS FOR SOLVING DIFFERENTIAL EQUATIONS AND THE CAUCHY PROBLEM

Annotation: The article is devoted to the consideration of various numerical methods for solving differential equations and the Cauchy problem. Namely: Euler's method, the Euler-Cauchy method (the corrected method), the improved Euler method, the Runge-Kutta classical method.

Key words: The Cauchy problem, the Euler method, the Runge-Kutta method, numerical methods, differential equations.

Many problems of mechanics, physics, and electrical engineering in mathematical modeling are reduced to solving differential equations.
A differential equation is an equation connecting the value of some unknown function at a certain point and the value of its derivatives of different orders at the same point.

The general form of the differential equation \( F(x, y, y') = 0 \), where \( y = y(x) \) is an unknown function of \( x \).

The normal form of the first-order differential equation is \( y' = F(x, y) \).

A differential equation of the form \( y' = F(x, y) \) in which the unknown function \( y \) depends on one argument \( x \) is called an ordinary differential equation, if several - by a partial differential equation.

The general solution of the differential equation \( y' = F(x, y) \) is the family of functions \( y = y(x, C) \). When solving applied problems, a particular solution is usually sought. The selection of a particular solution from the family of generic solutions is carried out using the initial conditions.

In classical analysis, a lot of methods for finding solutions of differential equations through elementary functions have been developed. Meanwhile, in solving practical problems, these methods are, as a rule, either completely useless, or their solution is associated with unacceptable costs of effort and time. To solve applied problems, methods for the approximate solution of differential equations are created, which can be conditionally divided into:

- Analytical methods, the application of which will solve differential equations in the form of an analytic function;
- Graphic methods, solving differential equations in the form of a graph;
- Numerical methods, when the desired function is obtained in the form of a table.

In practice, in most cases, it is not possible to find the exact solution of the mathematical problem that has arisen. This is mainly due not to the fact that we do not know how to do this, and since the solution sought is usually not expressed in the elementary or other known functions that are customary for us. Therefore, numerical methods became important, especially in connection with the growing
role of mathematical methods in various fields of science and technology and with the advent of high-performance computers.

Numerical methods are the main tool for solving complex mathematical models and problems. They reduce the solution of the problem to performing a finite number of arithmetic operations on the numbers and give the result in the form of a numerical value with an error acceptable for the given problem.

Within the framework of this article, numerical methods such as the Euler method, the Euler-Cauchy method (the corrected method), the improved Euler method, the classical Runge-Kutta method are considered.

**Euler's method**

The Euler method is a numerical method for obtaining the solution of a differential equation. The essence of Euler's method in step-by-step calculation of the values of the solution \( y = y(x) \) of a differential equation of the form \( y' = f(x, y) \) with the initial condition \((x_0; y_0)\).

The Euler method is a method of the first order of accuracy and is called the polygonal method. It plays an important role in the theory of numerical methods for solving ODE, although it is not often used in practical calculations due to low accuracy. The derivation of the calculated relations for this method can be performed in several ways: by geometric interpretation, using the Taylor expansion, of course by the difference method (using the difference approximation of the derivative) by a quadrature method (using an equivalent integral equation).

The main drawback of Euler's method is the systematic accumulation of errors.

**The Euler-Cauchy method (the corrected method)**

The essence of the corrected Euler method in step-by-step calculation of the values of the solution \( y = y(x) \) of a differential equation of the form \( y' = f(x, y) \) with the initial condition \((x_0; y_0)\). The Euler method is a second-order method of accuracy.
**Improved Euler method**

The improved Euler method is a numerical method for obtaining the solution of a differential equation.

The essence of the improved Euler method in step-by-step calculation of the values of the solution $y = y(x)$ of a differential equation of the form $y' = f(x, y)$ with the initial condition $(x_0; y_0)$.

The improved Euler method is a second-order precision method and is called the modified Euler method.

The Euler method and its modification considered above are essentially Runge-Kutta methods of the first and second order, respectively. Despite the increase in the amount of calculations, the fourth-order method has an advantage over the methods of the first and second orders, since it provides a small local error. This makes it possible to increase the integration step $h$ and, consequently, to shorten the calculation time.

**The classical Runge-Kutta method**

The classical Runge-Kutta method is one of the most well-known methods. This is a fourth-order accuracy method. The essence of the Runge-Kutta method in step-by-step calculation of the values of the solution $y = y(x)$ of a differential equation of the form $y' = f(x, y)$ with the initial condition $(x_0; y_0)$.

If we apply these methods in practice, we can come to the conclusion that the Runge-Kutta method of the fourth order gives the most accurate answer. In addition, it should be noted that the error (the discrepancy between the exact and approximate values) increases with each step of the calculation. This is due to the fact that, firstly, the approximate value obtained is rounded at each step, and secondly, by the fact that the value obtained in the previous step is taken as the basis of the calculation; approximation. Thus, an error is accumulated. With each new step, the approximate value is increasingly different from the exact one.
Использованные источники: